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DOCUMENT

OPS Angle Definition & Calculation

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1 INTRODUCTION

Purpose of this note is to describe the mathematical transformations, needed frames of reference and possible implementations used to compute the OPS angle with respect to the Earth Equator as defined, at high level, in [RD1].

While an orbital angle can be defined with respect to a generic XY plane of a reference system, here we will focus, according to [RD1], only on the definition of the OPS angle with respect to the Earth Fixed equatorial plane. This is important as the Earth equatorial plane is nominally not coincident with the XY plane of an inertial coordinate frame.

Note that the definitions and mathematical developments presented here are valid for an OPS angle defined with respect to the Earth Fixed equatorial plane, and they might not be valid for a different XY plane.

In Annex 1 (section 6) the frame of references involved into calculation of OPS angle are presented.

2 REFERENCE DOCUMENTS

[RD1] "Earth Observation OPS commanding", PE-TN-ESA-SY-0305, Version 1.9, 11-11-2011.

[RD2] "IERS Conventions (2010)", IERS Technical Note No. 36, 2010.

3 OPS ANGLE DEFINITION

Following the availability of GPS on-board, current and future ESA missions foresee an *on-board* schedule execution based on position in orbit (termed in fact Orbit Position Scheduling OPS) and not on time, which aims to precise execution of operation on specific ground location and removes the need for ground to have a precise orbital prediction.

Because the command execution is related to a ground target on the Earth's surface, the angle to be defined is preferred to be the one between the instantaneous Earth equatorial plane and the desired position in orbit; in this way the same angle (meaning same command) will correspond to the same ground location (for the same orbit in the repeat cycle) and in any case to same geographical latitude of the sub-satellite point assuming the inclination is identical.

In the literature the OPS angle is also commonly named True latitude, argument of latitude and/or orbit anomaly. A general definition of OPS angle is given in [RD1], if we constrain the OPS angle to be measured only with respect to the Earth equator, the following definition is applicable:

> ***OPS angle w.r.t. Earth Equator***
>
> *The OPS angle (α) with respect to the Earth equator is the angle defined in the <u>inertial and instantaneous</u> orbital plane between the <u>Earth equator</u> and the desired location on orbit for the command execution. The OPS angle is positive in the flight direction, and is measured from the ascending intersection of this orbital plane and the Earth equator.*

In other words it is the angle measured between the ascending node w.r.t the Earth equator (line of nodes) and the satellite's position vector.

Mathematically it can be defined as:

$$\cos(\alpha) = \frac{\vec{n} \cdot \vec{r}}{|\vec{n}| \cdot |\vec{r}|}$$

$$if\ \left(\hat{K} \cdot \vec{r}\right) < 0\ then\ \alpha = 2\pi - u$$

Where $\vec{r}$ is the position vector and $\vec{n}$ is the line of nodes, defined by

$$\vec{n} = \hat{K} \times \vec{h}$$

being $\widehat{K}$ the unitary vector perpendicular to the Earth equator, pointing North, and $\vec{h}$ the orbit angular momentum,

$$\vec{h} = \vec{r} \times \dot{\vec{r}}$$

3.1 Relation with Kepler elements.

It happens that the OPS angle can also be calculated as the sum of the osculating argument of perigee (measured w.r.t the earth Equator) and the osculating true anomaly.

$$\alpha = \omega + \nu$$

4 OPS ANGLE CALCULATION

In this section the formulas given in the section 3 are developed and reduced in order to provide the algorithm to calculate the OPS angle.

There is not a single way to calculate the OPS angle with respect to the equator, here three different developments of the algorithm, each one depending on the reference system in which state vector is given. Any other implementation remains valid as long as it is conform with the definition of OPS angle given above.

4.1 From a state vector given in an inertial frame

For simplicity we will name $\vec{\mathrm{r}}_{\mathrm{Inertial}} = (\mathrm{Rx}, \mathrm{Ry}, \mathrm{Rz})$ and $\dot{\vec{\mathrm{r}}}_{\mathrm{Inertial}} = (\mathrm{Vx}, \mathrm{Vy}, \mathrm{Vz})$, and all the sub-indices x,y,z refers to the vector expressed in the inertial reference system.

1. Calculate the unitary vector perpendicular to the Earth equator in the chosen inertial frame.

$$\hat{K} = (Kx, Ky, Kz) = [M]_{EF\ to\ INERTIAL}(0,0,1)^T$$

1. Calculate the angular momentum.

$$hx = Ry \cdot Vz - Rz \cdot Vy$$
$$hy = Rz \cdot Vx - Rx \cdot Vz$$
$$hz = Rx \cdot Vy - Ry \cdot Vx$$

2. Calculate the line of nodes.

$$nx = Ky \cdot hz - Kz \cdot hy$$
$$ny = Kz \cdot hx - Kx \cdot hz$$
$$nz = Kx \cdot hy - Ky \cdot hx$$

3. Calculate the orbit anomaly

$$\alpha = \cos^{-1}\left(\frac{\mathrm{Rx} \cdot \mathrm{nx} + \mathrm{Ry} \cdot \mathrm{ny} + \mathrm{Rz} \cdot \mathrm{nz}}{\sqrt{\mathrm{Rx}^2 + \mathrm{Ry}^2 + \mathrm{Rz}^2} \cdot \sqrt{\mathrm{nx}^2 + \mathrm{ny}^2 + nz^2}}\right)$$

$$\text{if } (\mathrm{Rx} \cdot \mathrm{Kx} + \mathrm{Ry} \cdot \mathrm{Ky} + \mathrm{Rz} \cdot \mathrm{Kz}) < 0 \text{ then } \alpha = 2\pi - \mathrm{u}$$

4.1.1 Note on Inertial Reference Systems

None of the Earth Centred reference systems is perfectly inertial, therefore when transforming properly from one reference system to another not only a transformation matrix must be used but also the derivative of the matrix to take into account the transformation of the velocity.

If the Earth Centred reference systems are considered inertial and used in the algorithm 4.1 presented above, then small differences in the OPS angle calculation (expected to be less than 0.25 micro degrees) may appear depending on the reference system used.

If a precise calculation is desired starting from an state vector expressed in Earth Centred reference system, then the residual velocity of the Earth Centred reference system with respect to inertial shall be removed from the state vector before applying the algorithm presented above.

4.2 From a state vector given in ECEqI

If the inertial frame given is the ECEqI the previous development can be simplified. The definition of this reference system is provided in Annex 1.

For simplicity we will name $\vec{r}_{ECEqI} = (Rx, Ry, Rz)$ and $\vec{\dot{r}}_{ECEqI} = (Vx, Vy, Vz)$

1. Calculate the line of nodes:

$$nx = (Rx \cdot Vz - Rz \cdot Vx)$$
$$ny = (Ry \cdot Vz - Rz \cdot Vy)$$

2. Calculate the orbit anomaly:

$$\alpha = \cos^{-1}\left(\frac{Rx \cdot nx + Ry \cdot ny}{\sqrt{Rx^2 + Ry^2 + Rz^2} \cdot \sqrt{nx^2 + ny^2}}\right)$$

$$\text{if } Rz < 0 \text{ then } \alpha = 2\pi - u$$

Or alternatively the osculating keplerian elements w and v can be calculated from the state vector given in ECEqI using the classical formulation and then the OPS angle can be calculated as

$$\alpha = \omega + \nu$$

4.3 From a state vector given in EF

If the state vector is given in the Earth fixed reference system two alternatives are proposed, depending on the accuracy required.

If high accuracy is desired, it is suggested to first perform a precise calculation transforming the state vector to any inertial reference system and then follow the algorithm of section 4.1.

If some tolerance is allowed, an approximate and simplified calculation is described, as its implementation is very simple. Initial test cases show the accuracy of this approximation in the order of micro-degrees.

Below both proposals are described in detail.

4.3.1 Precise Calculation

Because of the polar motion effect, the Earth rotation axis differs from the Earth Fixed reference system z-axis.

This effect makes the calculation of the OPS angle directly from a state vector expressed in the Earth Fixed reference system relatively complicated.

If a precise calculation of the OPS angle is required it is advised to transform from the earth Fixed reference system to any inertial reference system and proceed as per section 4.1.

NB: This algorithm is the one implemented in the Earth Observation Mission Software CFI version 4.4 released July 2012 where the EF state vector is transformed into TOD.

4.3.2 Simplified Calculation

When the state vector is given in Earth Fixed, the aim of calculating the OPS angle directly in this reference system is to avoid the use of transformation matrices and therefore simplify significantly the algorithms.

The simplification proposed considers that the angular velocity of the Earth is aligned with the Earth Fixed z-axis. Considering this approximation, the equations showed in section 3 are modified to account for the rotation velocity of the Earth, and the OPS angle is calculated directly from a state vector given in the Earth Fixed (EF) reference frame.

Replacing $\dot{\vec{r}} = (\dot{\vec{r}} + \omega \times \vec{r})$ to account for the angular velocity of the Earth, the formulas presented in section 3 are as follows:

Obtain the angular momentum

$$\vec{h} = \vec{r} \times (\dot{\vec{r}} + \omega \times \vec{r})$$

Calculate the line of nodes

$$\vec{n} = \widehat{K} \times \vec{h}$$

And finally calculate the orbit anomaly

$$\cos(\alpha) = \frac{\vec{n} \cdot \vec{r}}{|\vec{n}| \cdot |\vec{r}|}$$

$$\text{if } R_z < 0 \text{ then } \alpha = 2\pi - u$$

Developing these equations the following algorithms are obtained. For simplicity we will name $\vec{r}_{EF} = (Rx, Ry, Rz)$ and $\dot{\vec{r}}_{EF} = (Vx, Vy, Vz)$

1. Calculate the line of nodes:

$nx = (Rx \cdot Vz - Rz \cdot (Vx - \omega \cdot Ry))$
$ny = (Ry \cdot Vz - Rz \cdot (Vy + \omega \cdot Rx))$

2. Calculate the orbit anomaly:

$$\alpha = \cos^{-1}\left(\frac{Rx \cdot nx + Ry \cdot ny}{\sqrt{Rx^2 + Ry^2 + Rz^2} \cdot \sqrt{nx^2 + ny^2}}\right)$$

if $Rz < 0$ then $\alpha = 2\pi - u$

Being ω the earth velocity rotation = 0.000072921158553 rad / solar sec

Preliminary test cases show that the error on OPS angle committed when comparing this approximation with the precise calculation described in section 4.1 is in the order of one micro-degree.

5 OPS ANGLE RATE

In this section we will calculate the rate of the OPS angle, for this calculation two assumptions are made:

- The state vector is given in an inertial system and the RAAN is constant

Inertial system is a fair assumption for the True of Date and CIRS reference systems. The RAAN change in a small time interval is negligible for a sun-synchronous orbit.

These assumptions imply that the line of nodes is constant within an small interval of time,

$$(\mathrm{nx}, \mathrm{ny}, \mathrm{nz}) = \mathrm{cte}$$

- The radius of the orbit is constant

This is a fair assumption for a frozen LEO orbit, as the radius change in a small time interval is negligible. Therefore

$$\sqrt{\mathrm{Rx}^2 + \mathrm{Ry}^2 + \mathrm{Rz}^2} = cte$$

With this two assumptions, the following formula obtained in section 4.1 can be expressed as

$$\alpha = \cos^{-1}\left(\frac{\mathrm{Rx}\cdot\mathrm{nx} + \mathrm{Ry}\cdot\mathrm{ny} + \mathrm{Rz}\cdot\mathrm{nz}}{\sqrt{\mathrm{Rx}^2 + \mathrm{Ry}^2 + \mathrm{Rz}^2}\cdot\sqrt{\mathrm{nx}^2 + \mathrm{ny}^2 + nz^2}}\right) = \cos^{-1}\left(\frac{\mathrm{Rx}\cdot\mathrm{nx} + \mathrm{Ry}\cdot\mathrm{ny} + \mathrm{Rz}\cdot\mathrm{nz}}{cte}\right)$$

and its derivative will be.

$$\dot{\alpha} = \frac{-1}{\sqrt{1 - \left(\frac{\mathrm{Rx}\cdot\mathrm{nx} + \mathrm{Ry}\cdot\mathrm{ny} + \mathrm{Rz}\cdot\mathrm{nz}}{cte}\right)^2}} \cdot \left(\frac{\mathrm{Vx}\cdot\mathrm{nx} + \mathrm{Vy}\cdot\mathrm{ny} + \mathrm{Vz}\cdot\mathrm{nz}}{cte}\right)$$

And simplifying

$$\dot{\alpha} = \frac{-(\mathrm{Vx}\cdot\mathrm{nx} + \mathrm{Vy}\cdot\mathrm{ny} + \mathrm{Vz}\cdot\mathrm{nz})}{\sqrt{cte^2 - (\mathrm{Rx}\cdot\mathrm{nx} + \mathrm{Ry}\cdot\mathrm{ny} + \mathrm{Rz}\cdot\mathrm{nz})^2}}$$

6 ANNEX 1: ECEqI REFRENCE FRAME DEFINITION

The Earth Centred Equatorial Inertial Reference Frame (ECEqI) is defined by

Earth Centred Equatorial Inertial Reference Frame (ECEqI)

Origin: centre of the Earth
Fundamental plane: current Earth Equator

z-axis: points to the North Pole (IRP pole)
x-axis: points to the instantaneous position of the Greenwich meridian
y-axis: completes the right handed orthonormal reference system.

Note that:

- The position vector in the ECEqI reference frame is the same as in the Earth Fixed (EF) reference frame.

- The velocity vector in the ECEqI reference frame is the quasi-equivalent to the velocity vector in the Earth Fixed (EF) reference when corrected for the Earth rotation.

$$\vec{r}_{ECEqI} = \vec{r}_{EF}$$

$$\dot{\vec{r}}_{ECEqI} = \dot{\vec{r}}_{EF} + \vec{\omega} \times \vec{r}_{EF}$$

- The ECEqI reference frame is only <u>instantaneous</u> inertial. This coordinate frame is unsuitable for orbit propagation using e.g. the classical Kepler equations

6.1 Relationship between reference systems

Depending on the literature and the models used, different names are given to (quasi)equivalent reference frames. Here we will refer to:

- Inertial: equivalent to Mean of J2000.0, ECI, GCRS
- Earth Fixed: Equivalent to ITRS
- True of Date: Equivalent to CIRS, ERS

In the IERS Conventions [RD2] the transformation between the inertial and the Earth Fixed reference frames is defined, as well as the intermediate frames used, (green path in Fig.1)

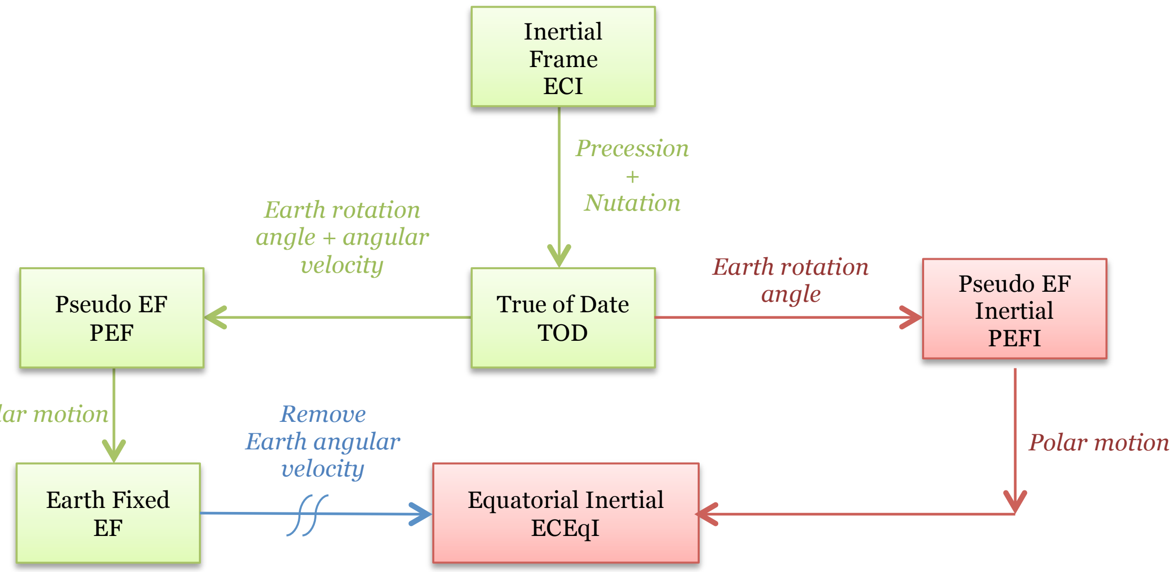

Figure 1: Transformations from Inertial to ECEqI

There is not a single path to transform from the inertial frame to the Equatorial Inertial ECEqI. Here two options are presented:

6.1.1 *Direct Path (red path in Fig.1)*

1. Transform from the Inertial to the True of Date reference frame as per described in IERS conventions [RD2]

2. Correct for the Earth rotation angle but not for the Earth angular velocity

$$\vec{r}_{PEFI} = R_3(\theta)\vec{r}_{TOD}$$

$$\dot{\vec{r}}_{PEFI} = R_3(\theta)\dot{\vec{r}}_{TOD}$$

   Where θ correspond to the ERA or the GAST angle, depending on the transformation used.

3. Apply the polar motion.

$$\vec{r}_{ECEqI} = [W]\vec{r}_{PEFI}$$

$$\dot{\vec{r}}_{ECEqI} = [W]\dot{\vec{r}}_{PEFI}$$

6.1.2 Indirect Path (blue path in Fig.1)

In some cases the transformation from the Inertial to the Earth fixed reference frame is already implemented and it is more convenient to add an extra step than to modify the implemented algorithm. Then the process would be,

1. Transform from the Inertial to the Earth Fixed reference frame as per described in IERS conventions [RD2]. (Already implemented)

2. Remove the Earth angular velocity. It shall be noted that the Earth angular velocity is aligned with the Z-axis of the ToD and the Pseudo Earth Fixed reference systems, therefore it shall be transformed into the Earth fixed reference system by applying the polar motion before to be subtracted from the velocity vector.

$$\vec{r}_{ECEqI} = \vec{r}_{EF}$$

$$\dot{\vec{r}}_{ECEqI} = \dot{\vec{r}}_{EF} + [W]\vec{\omega}\times\vec{r}_{EF}$$

2.1. Alternatively an approximation, similar to the one applied in section 4.3.2, is to consider that angular velocity of the Earth is aligned with the Earth Fixed z-axis, then the implementation becomes much simpler.

$$\vec{r}_{ECEqI} = \vec{r}_{EF}$$

$$\dot{\vec{r}}_{ECEqI} = \dot{\vec{r}}_{EF} + \vec{\omega}\times\vec{r}_{EF}$$

Here, as in section 4.3.2, the error committed is in the order of one micro-degree.